

# Where is quantum theory headed?

**Stephen L. Adler**

Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540, USA

E-mail: [adler@ias.edu](mailto:adler@ias.edu)

**Abstract.** Public talk at the EmQM13 conference opening event on “The Future of Quantum Mechanics”

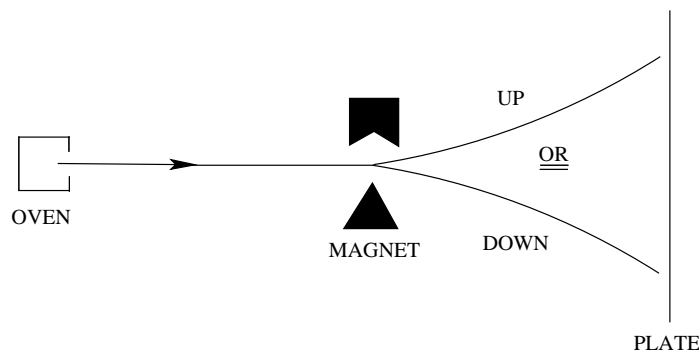
The organizers have asked me to state my views on the direction of the future development of quantum mechanics. Will it evolve within the standard framework, without the addition of new foundational physics? Or will the foundations require modification in an, at least in principle, experimentally detectable way?

First, let’s discuss the current status of quantum theory. Quantum mechanics is our most successful physical theory. It underlies our detailed understanding of atomic physics, chemistry, and nuclear physics, and the many technologies based on this knowledge. Additionally, relativistic quantum mechanics is the basis for the very successful standard model of elementary particles.

However, from its beginnings there have been conceptual problems associated with the nature of measurement in quantum mechanics. These can be simply illustrated with the famous Stern-Gerlach experiment (Fig. 1).

Silver atoms boiled off from a furnace are sent through a non-uniform magnetic field, and impinge on a photographic plate. Instead of a continuous distribution of spots, one sees two spots, corresponding to spin up and spin down relative to the magnetic field axis. Each atom

## STERN-GERLACH EXPERIMENT



**Figure 1.** Schematic representation of the Stern-Gerlach experiment

goes up OR down, but one cannot predict which in any given run – the results of the experiment are probabilistic. There is a 50% chance of an atom going up, and a 50% chance that it will go down.

From the point of view of the Schrödinger equation of quantum theory, this result has no explanation. In quantum theory, the state of the particle is described by its wave function, and the Schrödinger equation says that at a post-measurement final time  $T_f$ , the wave function is related to that at a pre-measurement initial time  $T_i$ , by a deterministic relation

$$\begin{aligned}\Psi(T_f) &= U(T_f, T_i)\Psi(T_i) \quad , \\ U(T_f, T_i) &= e^{iH(T_f - T_i)} \quad ,\end{aligned}$$

with the transition operator  $U$  completely specified by the Hamiltonian  $H$ . To explain what is observed, the Schrödinger equation must be supplemented by the reduction postulate and the Born rule. These state that the wave function only gives a description of probabilities when a measurement is made, with the probabilities for an “up” outcome and a “down” outcome given by the squares of the coefficients of the corresponding components in the initial wave function  $\Psi(T_i)$ ,

#### Born Rule for Probabilities

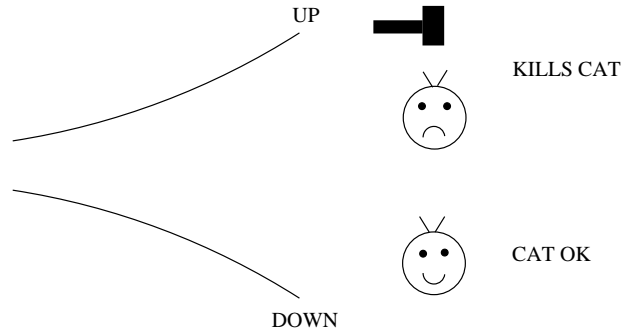
$$\begin{aligned}\Psi(T_i) &= C_{\text{up}}\Psi_{\text{up}} + C_{\text{down}}\Psi_{\text{down}} \quad , \\ \text{prob}_{\text{up}} &= |C_{\text{up}}|^2 \quad , \\ \text{prob}_{\text{down}} &= |C_{\text{down}}|^2 \quad , \\ |C_{\text{up}}|^2 + |C_{\text{down}}|^2 &= 1 \quad ,\end{aligned}$$

with the sum of the up and down probabilities equal to one. The reduction postulate and Born rule are an add-on to the Schrödinger equation. According to the Copenhagen interpretation of quantum mechanics, the Schrödinger equation applies when a microscopic system, the silver atom, is time-evolving in isolation. But when the atom interacts with a macroscopic measuring apparatus, as in the Stern–Gerlach setup, you have to use the reduction postulate and Born rule.

This situation leads to puzzles that have been debated for over eighty years. If quantum mechanics describes the whole universe, then why can’t one use the Schrödinger equation to describe the system consisting of the silver atom plus the measuring apparatus? But we never see a superposition state of the atom plus apparatus. This is Schrödinger’s famous cat paradox. Arrange the experiment so that an “up” outcome triggers a mechanism that kills the cat, while a “down” outcome keeps the cat alive. Of course we don’t do this, but if we were to do it, we would always see a live cat OR a dead one, never a superposition of the two (Fig. 2). So we have the problem of definite outcomes: where does the “either”–“or” dichotomy arise?

A related question is where do the probabilities come from? Quantum mechanics has probabilities without a sample space! An example of a sample space is a population of people, 40% blonde and 60% brunette. If you pick a person at random from the population, there is a probability  $\text{prob}_{\text{blonde}} = 0.4$  that you will have a blonde, and  $\text{prob}_{\text{brunette}} = 0.6$  that you will have a brunette. But the population (the sample space) is composed of individuals, with definite hair coloring – the probabilities only reflect our ignorance of details if we make a random pick without looking. Another example of a sample space, closer to our Stern–Gerlach experiment, is a coin toss. Consider 1000 coin tosses. If the coin is tossed without bias, you will find close to 500 heads and 500 tails, corresponding to  $\text{prob}_{\text{heads}} = 0.5$  and  $\text{prob}_{\text{tails}} = 0.5$ . Here the sample space consists of the 1000 detailed trajectories of the toss, which your eye cannot follow,

## SCHRÖDINGER CAT

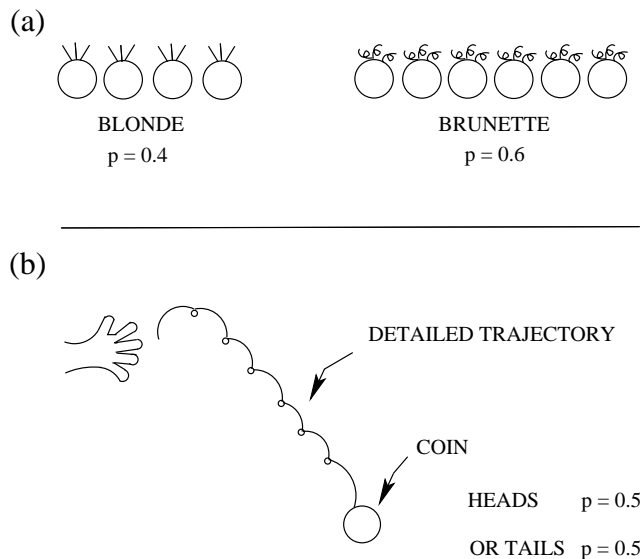


**Figure 2.** The Stern–Gerlach experiment with a Schrödinger cat as the outcome registration

but which if analyzed by a very fast computer could predict which toss would give a head and which a tail (Fig. 3). Again, the probabilities are just reflections of our ignorance of the details, but the details are there. So we have the questions – are there hidden details underlying the probabilities in quantum mechanics? Is there a hidden sample space?

Now we come to the question with which I began – where is future work to deal with these problems headed? Two routes proceed within quantum theory: (i) The first route within quantum theory is to try to change the interpretational rules. Examples are the so-called “many worlds” interpretation (all possibilities are there, we just only see one), and the so-called “histories” program, which sets up an observer-free generalization of the Copenhagen rules. (ii) The second route within quantum theory is to say there is a sample space, but we don’t see

## SAMPLE SPACE



**Figure 3.** Some sample spaces: (a) Populations of individuals with blonde/brunette hair coloring (b) Trajectories in a coin toss

it. One example is the proposal of Bohmian trajectories as the sample space underlying the Born rule. Another example comprises various statistical interpretations, which attribute the probabilistic outcomes to different internal states of the apparatus and or its environment, with the aim of deriving the Born rule.

None of these routes within orthodox quantum theory has gained general acceptance. Also, none makes experimental predictions at odds with standard theory, so experimentally they are not distinguishable.

The other possibility is to modify the foundations of quantum theory. Specifically, one gets a sample space by postulating additional degrees of freedom – so called “hidden variables”.

There are two possibilities for hidden variables – they can be *local*, or *nonlocal*. Local variables have the property that variables  $V(x_1, t_1)$  and  $W(x_2, t_2)$ , with  $x_1, x_2$  the spatial points and  $t_1, t_2$  the times of occurrence, cannot influence one another if the distance between them  $|x_1 - x_2|$  is greater than the distance  $c|t_1 - t_2|$  that light can travel, at velocity  $c$ , in the time interval from  $t_1$  to  $t_2$ . Such local variables are called causally separated. In quantum theory, causally separated variables have a commutative multiplication law

$$V(x_1, t_1) \times W(x_2, t_2) = W(x_2, t_2) \times V(x_1, t_1) \quad .$$

Ordinary numbers obey such a commutative law of multiplication, for example  $AB = BA$  for  $A = 7$  and  $B = 11$ , whereas in non-commutative multiplication, one would have  $AB \neq BA$ .

John Bell’s theorem asserts that local hidden variables plus the usual rules for probabilities imply certain inequalities that are not satisfied by quantum mechanical systems – and experiment sees that these inequalities are in fact violated. There is much discussion of possible loopholes, but I believe the result is robust, and that local hidden variables are excluded.

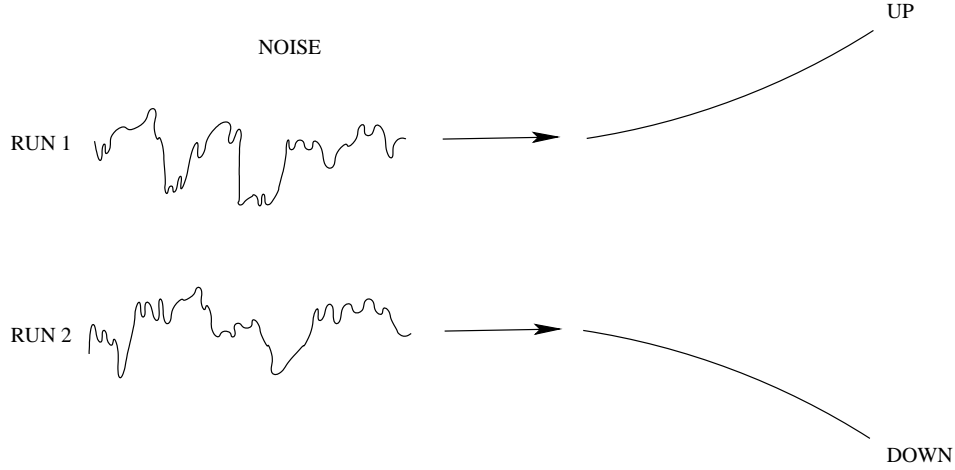
The other possibility is that the hidden variables are non-local: the hidden variables can act faster than the speed of light to establish correlations (as long as no faster than light signaling is possible). The hidden variables can also obey a non-commutative multiplication law.

For the rest of the talk I’ll focus on the possibility of non-local hidden variables – this is where my research interests lie. At a phenomenological level, there are very interesting models for the emergence of probabilities within the usual wave function formulation of nonrelativistic quantum theory, pioneered by Ghirardi, Rimini, and Weber in Trieste and by also by Pearle at Hamilton College in the U. S., and worked on by many others. These models postulate that space is filled with a very low level noise with a coupling to matter proportional to the imaginary unit  $i$ , rather than with a real-valued coupling (more technically, they couple through an anti-Hermitian Hamiltonian term). For example, there could be a small, rapidly fluctuating contribution to the gravitational potential or  $g_{00}$  metric component proportional to the imaginary unit  $i$ . If such a theory obeys two general properties, (1) the total probability of a particle being present remains one for all times (that is, the wave function normalization is preserved), and (2) there is no faster than light signaling, then the extra terms in the Schrödinger equation must have a special structure. This special structure allows one to *prove* definite outcomes obeying the Born rule!

In these models, for each repetition of the Stern–Gerlach experiment, the noise variable takes different values. For a large apparatus, these have a measurable effect, whereas for an atom not interacting with an apparatus, the effect is not measurable. The noise leads to different outcomes for different runs, with probabilities given by the Born rule. The different noises for different runs of the experiment are analogous, in the coin toss example I gave earlier, to different details of the tumbling coin trajectories for the different coin tosses (Fig. 4).

I’ve worked on phenomenological reduction models, but my main long term interest has been at the foundational level. I am trying to make an analogy between quantum mechanics emerging from a possible pre-quantum theory, and the known fact that thermodynamics emerges from the laws of statistical mechanics. Thermodynamics – the science of heat and work (Fig. 5) – reflects

## PHENOMENOLOGICAL REDUCTION MODELS



**Figure 4.** Different noise histories, in objective reduction models, can explain “up” and “down” registrations in the Stern–Gerlach experiment

averaged properties of huge numbers of atoms. It is a complete, consistent system by itself, and remarkably was discovered in the 19th century *before* the existence of atoms was established. But from statistical mechanics – the laws of motion of large systems of atoms, one can deduce the laws of thermodynamics, together with details of fluctuation corrections to thermodynamics, so called Brownian motion corrections. (The figure shows the random walk trajectory of a pollen grain being bombarded by molecules in thermal motion.)

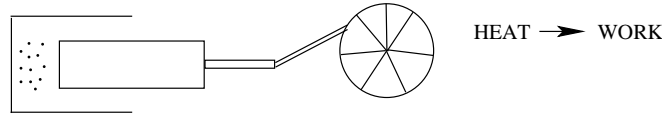
My suggestion, in articles with collaborators and a small book that I wrote in 2004 [1] , is a theory that I call “trace dynamics”. It is a classical-like system of non-commutating variables – even distant systems in the universe are interacting instantaneously with us. One can make sense of the mathematics of non-commuting variables by using the cyclic property of a mathematical operation called the Trace:  $\text{Trace}ABCD..FG = \text{Trace}GABCD..F = \text{Trace}FGBCD...$  This is a very powerful tool. One can use it to set up a system of equations analogous to classical mechanics, and to do statistical averaging. Getting a little technical now, for those in the audience familiar with quantum theory and statistical physics, what distinguishes trace dynamics from ordinary classical mechanics is the existence of a generic conserved quantity in addition to the energy and momentum. This quantity is operator-valued, and has the form

$$\begin{aligned} \text{Conserved operator in trace dynamics} = \\ \sum_{\text{bosons}} [q_{\text{boson}} p_{\text{boson}} - p_{\text{boson}} q_{\text{boson}}] \\ - \sum_{\text{fermions}} [q_{\text{fermion}} p_{\text{fermion}} + p_{\text{fermion}} q_{\text{fermion}}] \quad , \end{aligned}$$

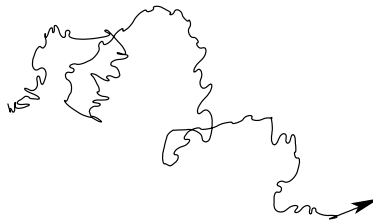
with the  $qs$  the canonical coordinates and the  $ps$  the canonical momenta. This is reminiscent in structure to the canonical commutation and anti-commutation relations of quantum theory. Just as energy in statistical mechanics is equally partitioned between the various degrees of freedom, one might expect this conserved operator, in a statistical mechanical treatment of trace dynamics, to also be equi-partitioned, giving the starting point for quantum theory.

My conjectures thus are: statistical averages in trace dynamics give the Schrödinger equation

(a) THERMODYNAMICS



(b) BROWNIAN MOTION



**Figure 5.** (a) Thermodynamics: the science of heat and work (b) Brownian motion of a pollen grain being bombarded by molecules in the liquid in which it is suspended

and operator algebra of quantum theory, while Brownian motion corrections give the low level noise on which phenomenological reduction models are based. I talked about this program in my keynote address at the Vienna conference two years ago. Currently I am working on incorporating gravity into trace dynamics [2], and that is what I will talk about tomorrow. My approach to an emergent quantum theory is still a work in progress – there is much yet to be done!

### Acknowledgments

I wish to thank Gerhard Grössing for organizing the conference EmQM13 on Emergent Quantum Mechanics, the Fetzer Franklin fund for its financial support, and Susan Higgins for redrawing my figure sketches for publication.

### References

- [1] S.L. Adler *Quantum Theory as an Emergent Phenomenon: The Statistical Mechanics of Matrix Models as the Precursor of Quantum Field Theory*, Cambridge University Press, Cambridge (2004).
- [2] S. L. Adler, “Incorporating gravity into trace dynamics: the induced gravitational action”, arXiv:1306.0482, Class. Quantum Grav. 30 (2013) 195015 and Class. Quantum Grav. 30 (2013) 239501 (Erratum); S. L. Adler and F. R. Ramazanoğlu, “Spherically symmetric vacuum solutions arising from trace dynamics modifications to gravitation”, arXiv:1308.1448.